Multiperfect Numbers

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Little is currently known about multiperfect numbers of abundancy greater than 2. It has been conjectured that all multiperfect numbers of abundancy between 3 and 7 have been discovered. If this is true, no 3-multiperfect numbers greater than 10^{300} exist (the largest one known is 51001180160), and thus no odd perfect numbers exist. It has also been proven by Lehmer that a 3-multiperfect number must have at least three distinct prime factors.

We have already shown that it is trivial to prove that twice any odd perfect number will be 3-multiperfect. However, we may generalize this theorem to find new multiperfect numbers from existing ones.

Theorem: Let n be a p-multiperfect number, with $k \geq 2$ and prime p. If n is not divisible by p, pn is $p+1$ multiperfect.

Proof:

Since $p \nmid n$ and p is prime, p and n are relatively prime. Therefore, we may write $\sigma(pn)$ as $\sigma(p) * \sigma(n)$. Recall that $\sigma(p) = p + 1$ for all primes. Since $\sigma(n)$ is p-multiperfect, by definition $\sigma(n) = pn$. Thus,
$\sigma(pn) = \sigma(p) * \sigma(n) = (p + 1)(pn)$, which indicates that pn is $p + 1$ multiperfect.

This may suggest that infinitely many multiperfect numbers exist (irrespective of the conjecture that infinitely many perfect numbers, and thus multiperfects of abundancy 2, exist), as infinitely many primes exist. However, the behavior of multiperfect numbers in this respect for different multiplicities is highly unpredictable and more research is needed before this question may be approached.