The Divisor Function, $\sigma(n)$.

5th Annual STE Research Conference - April 12, 2006

While $\sigma(n)$ can be found directly by summing the divisors of n, there are several shortcuts that will allow us to compute the value of the function more quickly:

- If n is prime, n will have only the divisors 1 and n by definition. Therefore, $\sigma(n) = 1 + n$ and $\tau(n) = 2$.
- If n is a power of a prime, p^α, $\sigma(n) = \frac{p^{\alpha+1}-1}{p-1}$ and $\tau(n) = \alpha + 1$.
- We can redefine any natural number n as the product of its primes. Because $\sigma(n)$ is multiplicative and prime numbers are all relatively prime to each other, we can define $\sigma(n)$ as:

  $$\sigma(n) = \sigma(p_1^{\alpha_1}) * \sigma(p_2^{\alpha_2}) * ... = \prod_i \frac{p_i^{\alpha_i+1} - 1}{p_i - 1}$$

  where p_1, p_2, ... are prime factors of n and α_1,α_2, etc. are the multiplicities of those factors.
  Similarly, we can define $\tau(n)$ as:

  $$\prod_i (\alpha_i + 1)$$

- If n is a perfect square or twice a perfect square, $\sigma(n)$ will be odd. If n is a perfect square, $\tau(n)$ will be odd. In all other cases, $\sigma(n)$ and $\tau(n)$ are even.