Definitions

5th Annual STE Research Conferenece - April 12, 2006

We define the divisor function, $\sigma_k(n)$, as $\sum_{d|n} d^k$, the sum of the k-th powers of all divisors of n.

When the k is omitted, we assume it to be one. That is, $\sigma(n) = \sum_{d|n} d$.

We use the notation $\tau(n)$ to denote the number-of-divisors function. This is equivalent to the cardinality of the set of divisors and is also equivalent to $\sigma_0(n)$, since all divisors to the 0th power become ones.

The divisor function (for all subscripts) is **multiplicative**:
$(\exists m, c \in \mathbb{N})\,(n = mc \wedge \gcd(m,c) = 1) \rightarrow (\sigma(n) = \sigma(mc) = \sigma(m) * \sigma(c))$. In other words, if a number can be decomposed into two relatively prime factors, the sum of its divisors is that of each of the two factors mulitplied together.

A **perfect number** is a natural number n such that $\sigma(n) = 2n$.

A ***k*-perfect number** is a multiperfect number of class k, or a natural number n such that $\sigma(n) = kn$.

The **Ore harmonic number**, $H(n)$, is defined by Oystein Ore as:

$$H(n) = \frac{\tau(n)}{\sum_{d|n} \frac{1}{d}}$$

However, we will later prove the identity:

$$H(n) = \frac{\tau(n)}{\sum_{d|n} \frac{1}{d}} = \frac{n * \tau(n)}{\sigma(n)}$$

Which is useful, since the latter form is easier to work with.